Period 4, April 15, 2025

$2^2 = 4$
$3^2 = 9$
$4^2 = 16$
$5^2 = 25$
$6^2 = 36$

5, 7, 9, 11

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, ...

1,	1,	2,	3,	5,	8.
$a_1 = 1$	$a_2 = 1$	$a_3 = 2$	$a_4 = 3$	$a_5 = 5$	$a_6 = 8$

The notation a_n represents the nth term, or **general term**, of a sequence.

Definition of a Sequence

$a_1, a_2, a_3, a_4 \ldots$

An **infinite sequence** $\{a_n\}$ is a function whose domain is the set of positive integers. The function values, or **terms**, of the sequence are represented by

$$a_1, a_2, a_3, a_4, \ldots, a_n, \ldots .$$

Sequences whose domains consist only of the first n positive integers are called **finite sequences**.

a_1, a_2, a_3, a_4

Write the first four terms of the sequence whose nth term, or general term, is given:

a. $a_n = 3n + 4$

$a_1 = 3(1) + 4 = 7$

$a_2 = 3(2) + 4 = 10$

$a_3 = 3(3) + 4 = 13$

$a_4 = 3(4) + 4 = 16$

$7, 10, 13, 16$

b. $a_n = \dfrac{(-1)^n}{3^n - 1}$.

$a_1 = \dfrac{(-1)^1}{3^1 - 1} = \dfrac{-1}{3-1} = -\dfrac{1}{2}$

$a_2 = \dfrac{(-1)^2}{3^2 - 1} = \dfrac{1}{9-1} = \dfrac{1}{8}$

$a_3 = \dfrac{(-1)^3}{3^3 - 1} = \dfrac{-1}{27-1} = \dfrac{-1}{26}$

$a_4 = \dfrac{(-1)^4}{3^4 - 1} = \dfrac{1}{81-1} = \dfrac{1}{80}$

$-\dfrac{1}{2}, \dfrac{1}{8}, -\dfrac{1}{26}, \dfrac{1}{80}$

as a function of n, the number of the term, sequences can also be defined using **recursion formulas**. A recursion formula defines the nth term of a sequence as a function of the previous term. Our next example illustrates that if the first term of a sequence is known, then the recursion formula can be used to determine the remaining terms.

Find the first four terms of the sequence in which $a_1 = 5$ and $a_n = 3a_{n-1} + 2$ for $n \geq 2$.

$a_1 = \underline{5}$ $\qquad a_n = 3(a_{n-1}) + 2$

$a_2 = 3(a_{2-1}) + 2 = 3a_1 + 2 = 3 \cdot 5 + 2 = \underline{17}$

$a_3 = 3(a_{3-1}) + 2 = 3a_2 + 2 = 3(17) + 2 = \underline{36}$

$a_4 = 3(a_{4-1}) + 2 = 3a_3 + 2 = 3(36) + 2 = 110$

$5, 17, 36, 74$

Factorial Notation

If n is a positive integer, the notation $n!$ (read "n factorial") is the product of all positive integers from n down through 1.

$$n! = n(n-1)(n-2)\cdots(3)(2)(1)$$

0! (zero factorial), by definition, is 1.

$$0! = 1$$

$1! = 1$

$2! = 2 \cdot 1 = 2$

$3! = 3 \cdot 2 \cdot 1 = 6$

$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

$n! = n(n-1)(n-2)(n-3)\cdots 3 \cdot 2 \cdot 1$

$(n+1)! = (n+1)(n+1-1)(n+1-2)(n+1-3)\cdots 3 \cdot 2 \cdot 1$
$= (n+1)(n)(n-1)(n-2)\cdots 3 \cdot 2 \cdot 1$

$(n-2)! = (n-2)(n-2-1)(n-2-2)(n-2-3)\cdots 3 \cdot 2 \cdot 1$
$= (n-2)(n-3)(n-4)\cdots 3 \cdot 2 \cdot 1$

the first four terms of the sequence whose nth term is

$$a_n = \frac{2^n}{(n-1)!}.$$

$a_1 = \frac{2^1}{(1-1)!} = \frac{2}{0!} = \frac{2}{1} = 2$

$a_2 = \frac{2^2}{(2-1)!} = \frac{4}{1!} = \frac{4}{1} = 4$

$a_3 = \frac{2^3}{(3-1)!} = \frac{8}{2!} = \frac{8}{2\cdot 1} = \frac{8}{2} = 4$

$a_4 = \frac{2^4}{(4-1)!} = \frac{16}{3!} = \frac{16}{3\cdot 2\cdot 1} = \frac{16}{6} = \frac{8}{3}$

$\left(2, 4, 4, \frac{8}{3}\right)$

CHECK POINT 3 Write the first four terms of the sequence whose nth term is

$$a_n = \frac{20}{(n+1)!}.$$

$a_1 = \frac{20}{(1+1)!} = \frac{20}{2!} = \frac{20}{2\cdot 1} = \frac{20}{2} = 10$

$a_2 = \frac{20}{(2+1)!} = \frac{20}{3!} = \frac{20}{3\cdot 2\cdot 1} = \frac{20}{6} = \frac{10}{3}$

$a_3 = \frac{20}{(3+1)!} = \frac{20}{4!} = \frac{20}{4\cdot 3\cdot 2\cdot 1} = \frac{20}{24} = \frac{5}{6}$

$a_4 = \frac{20}{(4+1)!} = \frac{20}{5!} = \frac{20}{5\cdot 4\cdot 3\cdot 2\cdot 1} = \frac{20}{120} = \frac{1}{6}$

$\left(10, \frac{10}{3}, \frac{5}{6}, \frac{1}{6}\right)$

Evaluate each factorial expression:

a. $\frac{10!}{2!8!}$ **b.** $\frac{(n+1)!}{n!}$.

$$\frac{10!}{2!\cdot 8!} = \frac{10\cdot 9\cdot \cancel{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}}{2\cdot 1\cdot \cancel{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}} = \frac{90}{2} = 45$$

$$\frac{(n+1)!}{n!} = \frac{(n+1)\cancel{n(n-1)(n-2)\cdots 3\cdot 2\cdot 1}}{\cancel{n(n-1)(n-2)\cdots 3\cdot 2\cdot 1}} = n+1$$

Summation Notation

The sum of the first n terms of a sequence is represented by the **summation notation**

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + a_4 + \cdots + a_n,$$

where i is the **index of summation**, n is the **upper limit of summation**, and 1 is the **lower limit of summation**.

Expand and evaluate the sum:

a. $\sum_{i=1}^{6}(i^2+1)$ **b.** $\sum_{k=4}^{7}[(-2)^k - 5]$ **c.** $\sum_{i=1}^{5}3$. $\sum_{i=1}^{6}2i^2$

$$\sum_{i=1}^{6}(i^2+1) = (1^2+1)+(2^2+1)+(3^2+1)+(4^2+1)+(5^2+1)+(6^2+1)$$
$$= 2+5+10+17+26+37 = 97$$

$$\sum_{k=4}^{7}[(-2)^k - 5] = ((-2)^4-5)+((-2)^5-5)+((-2)^6-5)+((-2)^7-5)$$
$$(16-5)+(-32-5)+(64-5)+(-128-5) = 11-37+59-133 = -100$$

$$\sum_{i=1}^{5}3 = 3+3+3+3+3 = 15$$

$$\sum_{i=1}^{6}2(i^2) = 2(1)^2+2(2)^2+2(3)^2+2(4)^2+2(5)^2+2(6)^2$$
$$= 2\cdot 1+2\cdot 4+2\cdot 9+2\cdot 16+2\cdot 25+2\cdot 36$$
$$2(1+4+9+16+25+36) = 2\cdot 91 = 182$$

Property	Example
1. $\sum_{i=1}^{n} ca_i = c\sum_{i=1}^{n} a_i$, c any real number	$\sum_{i=1}^{4} 3i^2 = 3\cdot 1^2 + 3\cdot 2^2 + 3\cdot 3^2 + 3\cdot 4^2$ $3\sum_{i=1}^{4} i^2 = 3(1^2 + 2^2 + 3^2 + 4^2) = 3\cdot 1^2 + 3\cdot 2^2 + 3\cdot 3^2 + 3\cdot 4^2$ Conclusion: $\sum_{i=1}^{4} 3i^2 = 3\sum_{i=1}^{4} i^2$
2. $\sum_{i=1}^{n} (a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$	$\sum_{i=1}^{4} (i + i^2) = (1 + 1^2) + (2 + 2^2) + (3 + 3^2) + (4 + 4^2)$ $\sum_{i=1}^{4} i + \sum_{i=1}^{4} i^2 = (1 + 2 + 3 + 4) + (1^2 + 2^2 + 3^2 + 4^2)$ $= (1 + 1^2) + (2 + 2^2) + (3 + 3^2) + (4 + 4^2)$ Conclusion: $\sum_{i=1}^{4} (i + i^2) = \sum_{i=1}^{4} i + \sum_{i=1}^{4} i^2$
3. $\sum_{i=1}^{n} (a_i - b_i) = \sum_{i=1}^{n} a_i - \sum_{i=1}^{n} b_i$	$\sum_{i=3}^{5} (i^2 - i^3) = (3^2 - 3^3) + (4^2 - 4^3) + (5^2 - 5^3)$ $\sum_{i=3}^{5} i^2 - \sum_{i=3}^{5} i^3 = (3^2 + 4^2 + 5^2) - (3^3 + 4^3 + 5^3)$ $= (3^2 - 3^3) + (4^2 - 4^3) + (5^2 - 5^3)$ Conclusion: $\sum_{i=3}^{5} (i^2 - i^3) = \sum_{i=3}^{5} i^2 - \sum_{i=3}^{5} i^3$

$\sum_{k=1}^{4} k(k+4) = \square$ (Simplify your answer.)

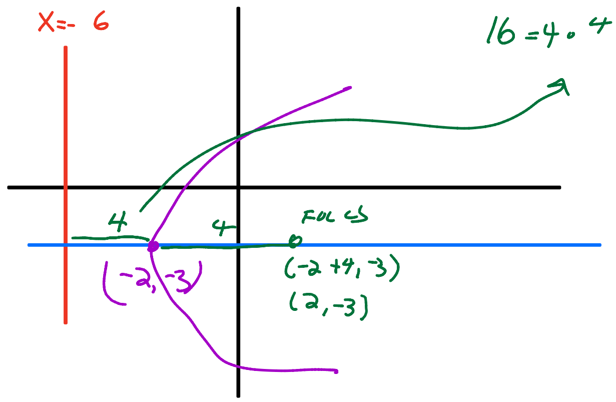
$1(1+4)+2(2+4)+3(3+4)+4(4+4)$

$1\cdot 5+2\cdot 6+3\cdot 7+4\cdot 8$

$5+12+21+28=66$

Latus = 4 · Focal Length

$16 = 4 \cdot 4$



Equation

$(y-k)^2 = +4p(x-h)$

$(y-(-3))^2 = 4 \cdot 4(x-(-2))$

$(y+3)^2 = 16(x+2)$

II $x^2 - 8x - 4y + 8 = 0$

$x^2 - 8x + 16 = 4y - 8 + 16 \Rightarrow (x-4)^2 = 4y + 8 \Rightarrow (x-4)^2 = 4(y+2)$

$a = 1$

$b = -8$

$\frac{b}{2} = \frac{-8}{2} = -4$

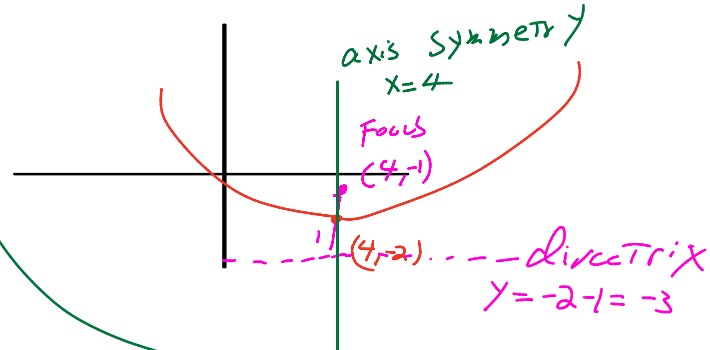
$\left(\frac{b}{2}\right)^2 = (-4)^2 = 16$

$(x-4)^2 = 4(y+2)$ opens up

Vertex $(4, -2)$

Focal Length $= 1$

Latus $= 4$



III

Vertex (2, -1)

directrix ⇒ $x = -5$

Focus $(2+7, -1)$

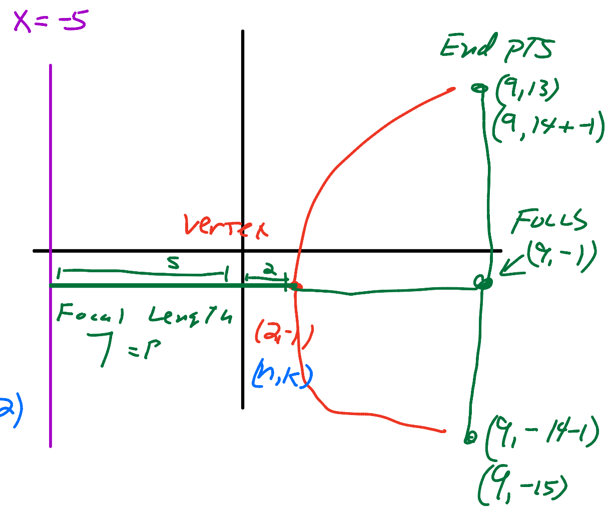
$(9, -1)$

Latus Length $= 4 \cdot 7 = 28$

Equation Right $(y-k)^2 = 4p(x-h)$

$(y-(-1))^2 = 4 \cdot 7(x-2)$

$(y+1)^2 = 28(x-2)$



Solve the equation on the interval $0 \le \theta < 2\pi$.

19) $(\csc\theta - 2)(\cot\theta + 1) = 0$

A) $\left\{\frac{\pi}{6}, \frac{3\pi}{4}, \frac{5\pi}{6}, \frac{5\pi}{4}\right\}$

B) $\left\{\frac{\pi}{6}, \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{6}\right\}$

C) $\left\{\frac{3\pi}{4}, \frac{7\pi}{6}, \frac{5\pi}{4}, \frac{11\pi}{6}\right\}$

D) $\left\{\frac{\pi}{6}, \frac{3\pi}{4}, \frac{5\pi}{6}, \frac{7\pi}{4}\right\}$

Find the focus and directrix of the parabola with the given equation. Then graph the parabola.

$x^2 = 20y \Rightarrow (x-0)^2 = 20(y-0)$ vertex $= (0,0)$

opens up

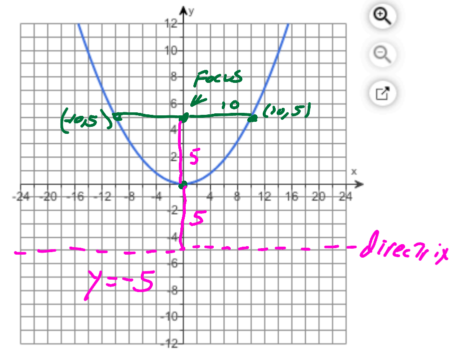
The focus is (0,5).
(Type an ordered pair.)

The directrix is $y = -5$.
(Type an equation.)

Focal Length $\frac{20}{4} = 5$

Latus Length $= 20$

Use the graphing tool to graph the parabola only.



Convert the equation to standard form by completing the square on x and y. Then, graph the hyperbola. Locate the foci and find the equations of the asymptotes.

$16x^2 - y^2 - 64x - 10y + 43 = 0$
$\quad -43 \quad -43$

The standard form of the equation is $\square$.
(Type an equation. Use integers or fractions for any numbers in the expression.)

$16x^2 - 64x \quad -y^2 - 10y \quad = -43$

$16(x^2 - 4x + 4) - 1(y^2 + 10y + 25) = -43 + 4 \cdot 16 + -1 \cdot 25$

$a = 1$, $b = -4$, $\frac{b}{2} = \frac{-4}{2} = -2$, $\left(\frac{b}{2}\right)^2 = (-2)^2 = 4$

$a = 1$, $b = 10$, $\frac{b}{2} = \frac{10}{2} = 5$, $\left(\frac{b}{2}\right)^2 = (5)^2 = 25$

$16(x-2)^2 - 1(y+5)^2 = -43 + 64 - 25$

$\frac{16(x-2)^2 - (y+5)^2}{-4} = \frac{-4}{-4} \Rightarrow -4(x-2)^2 + \frac{(y+5)^2}{4} = 1$

$\frac{(y+5)^2}{4} - \frac{(x-2)^2}{\frac{1}{4}} = 1$

Centers $(2, -5)$

Up/Down

$\sqrt{4} = 2$

$\sqrt{\frac{1}{4}} = \frac{1}{2}$

$F^2 = 2^2 + \left(\frac{1}{2}\right)^2$

$F^2 = 4 + \frac{1}{4} = 4\frac{1}{4} = \frac{17}{4}$

$F = \frac{\sqrt{17}}{2}$ = Focal Length

Point $(2, -5)$

$y - (-5) = -4(x - 2)$

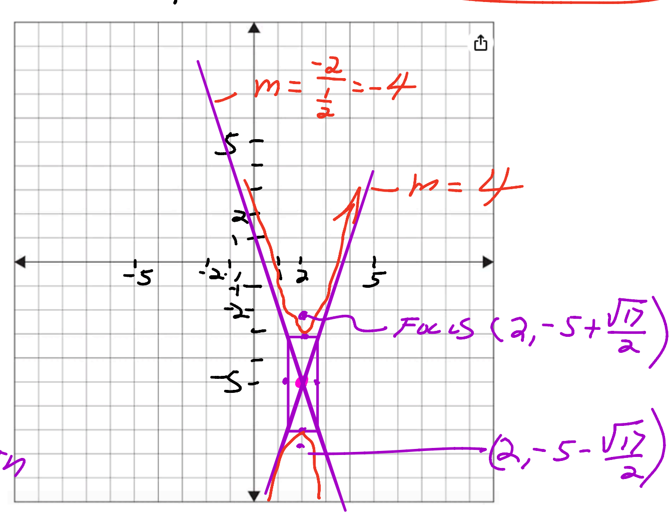
$y + 5 = -4x + 8$
$\quad -5 \qquad\qquad -5$

$y = -4x + 3$

$y - (-5) = 4(x - 2)$

$y + 5 = 4x - 8$
$\quad -5 \qquad\quad -5$

$y = 4x - 13$

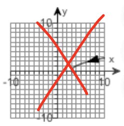


Use point plotting to graph the plane curve described by the given parametric equations. Use arrows to show the orientation of the curve corresponding to increasing values of t.

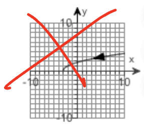
$x = t - 3, \quad y = \sqrt{t}; \quad t \ge 0$

Choose the correct graph below.

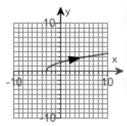
A.



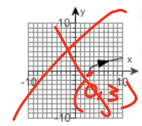
B.



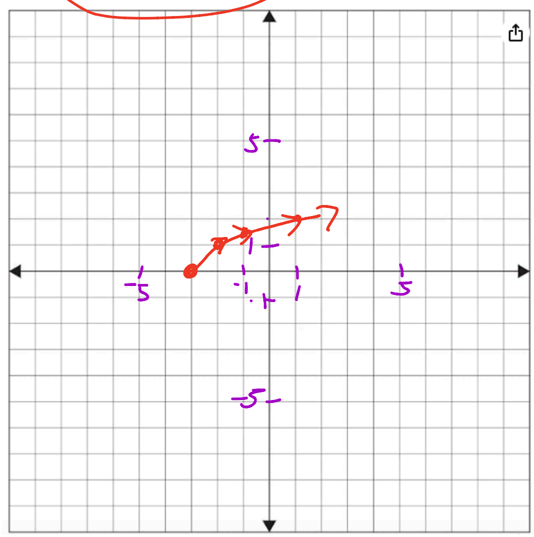
C.



D.



T	$X = T-3$	$Y = \sqrt{T}$	Point
0	$-3 = 0-3$	$0 = \sqrt{0}$	$(-3,0)$
1	$-2 = 1-3$	$1 = \sqrt{1}$	$(-2,1)$
2	$-1 = 2-3$	$1.4 = \sqrt{2}$	$(-1,1.4)$
3			
4	$1 = 4-3$	$2 = \sqrt{4}$	$(1,2)$



Use the vertex and the direction in which the parabola opens to determine the relation's domain and range. Is the relation a function?

$y^2 + 8y - x + 12 = 0$

$+x - 12 \quad +x-12$

The domain is $[-4,\infty)$. (Type your answer in interval notation.)

The range is $(-\infty,\infty)$. (Type your answer in interval notation.)

Is the relation a function?

- Yes
- No

$y^2 + 8y + 16 = x - 12 + 16$

$a = 1$

$b = 8$

$\left(\frac{b}{2}\right) = \frac{8}{2} = 4$

$\left(\frac{b}{2}\right)^2 = (4)^2 = 16$

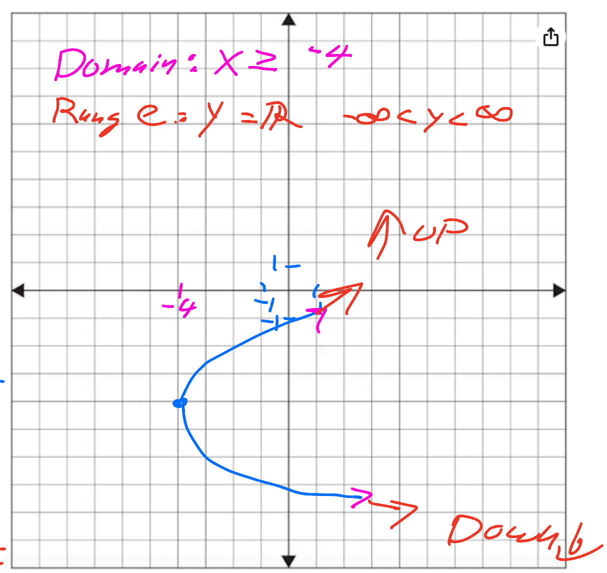
opens Right

$(y+4)^2 = 1(x+4)$

vertex $(-4,-4)$

LaTus $= 1$

Focal Length $= \frac{1}{4}$



Convert the equation to standard form by completing the square on x and y. Then graph the ellipse and give the location of its foci.

$16x^2 + 25y^2 - 64x + 50y - 311 = 0$

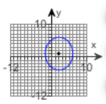
$+311 \quad +311$

The standard form of the equation is $\frac{(x-2)^2}{25} + \frac{(y+1)^2}{16} = 1$.

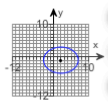
(Type an equation. Simplify your answer.)

Choose the correct graph below.

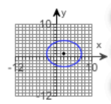
A.



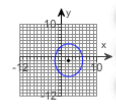
B.



C.



D.



The foci are located at $(5,-1),(-1,-1)$.

(Type ordered pairs. Use a comma to separate answers as needed. Simplify your answers. Type an exact answer, using radicals as needed.)

$16x^2 - 64x \quad + 25y^2 + 50y \quad = 311$

$16(x^2 - 4x + 4) + 25(y^2 + 2y + 1) \quad = 311 + 4 \cdot 16 + 25 \cdot 1$

$= 311 + 64 + 25$

$= 311 + 89$

$= 400$

$a = 1$ $\quad$ $a = 1$

$b = -4$ $\quad$ $b = 2$

$\frac{b}{2} = \frac{-4}{2} = -2$ $\quad$ $\frac{b}{2} = \frac{2}{2} = 1$

$\left(\frac{b}{2}\right)^2 = (-2)^2 = 4$ $\quad$ $\left(\frac{b}{2}\right)^2 = (1)^2 = 1$

$\frac{16(x-2)^2 + 25(y+1)^2}{400} = \frac{400}{400}$

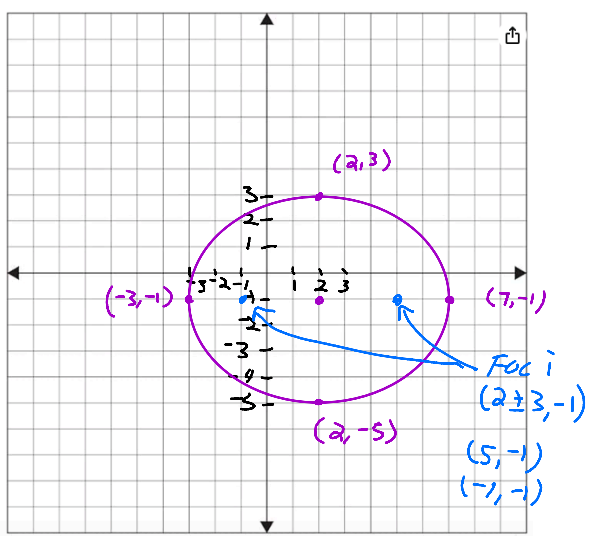
$\frac{16(x-2)^2}{400} + \frac{25(y+1)^2}{400} = 1$

$\frac{(x-2)^2}{25} + \frac{(y+1)^2}{16} = 1$

center $(2,-1)$

semi major $= x : \sqrt{25} = 5$

semiminor $= y = \sqrt{16} = 4$



$F^2 + 4^2 = 5^2$

$F^2 + 16 = 25$

$F^2 = 9$

$F = 3, -3$

Find the solution set for each system by graphing both of the system's equations in the same rectangular coordinate system and finding points of intersection. Check all solutions in both equations.

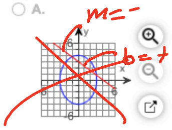
$$\begin{cases} 9x^2 + 16y^2 = 144 \Rightarrow \text{Ellipse} \\ 3x - 4y = 12 \Rightarrow \text{Line} \end{cases}$$

$9(0)^2 + 16(-3)^2 = 144$ True | $9(4^2) + 16(0)^2 = 144$ True

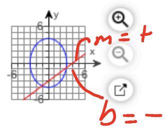
$3(0) - 4(-3) = 12$ True | $3(4) - 4(0) = 12$ True

Graph both of the system's equations in the same rectangular coordinate system. Choose the correct graph below.

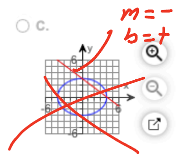
A.



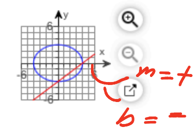
B.



C.



D.



Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. The solution set is $\{(0,-3),(4,0)\}$.
(Type an ordered pair. Use a comma to separate answers as needed.)

B. There is no solution.

$$\frac{9x^2}{144} + \frac{16y^2}{144} = \frac{144}{144}$$

$$\frac{9x^2}{144} + \frac{16y^2}{144} = 1$$

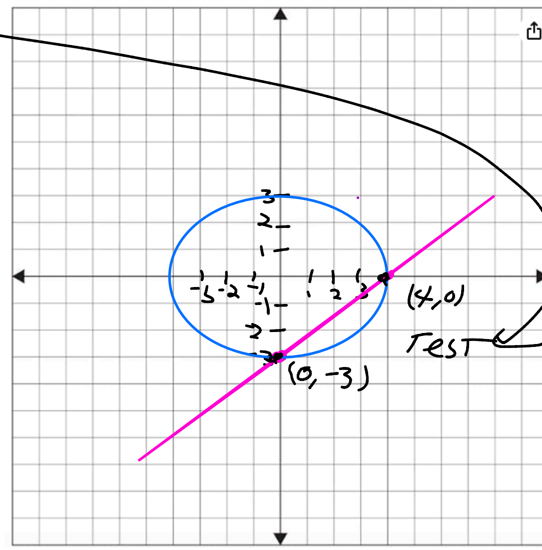
$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

Semimajor $= 4$: x-axis

Semi minor $= 3$: y-axis

$$3x - 4y = 12$$
$$-3x \qquad -3x$$
$$\frac{-4y}{-4} = \frac{-3x + 12}{-4}$$
$$y = \frac{-3x}{-4} + \frac{12}{-4}$$
$$y = \frac{3}{4}x - 3$$

$m = \frac{3}{4}$ $\quad b = -3$



Find the standard form of the equation of the hyperbola satisfying the given conditions.

Foci at (− 3,0) and (3,0); vertices at (1,0) and (− 1,0)

The equation is $x^2 - \frac{y^2}{8} = 1$.

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Center (0,0)
opens Left/Right

$F^2 = 1^2 + b^2$
$3^2 = 1^2 + b^2$
$9 = 1 + b^2$
$8 = b^2$
$\pm 2\sqrt{2} = b = \sqrt{8}$

$$\frac{(x-0)^2}{1^2} - \frac{(y-0)^2}{(\sqrt{8})^2} = 1$$

$$x^2 - \frac{y^2}{8} = 1$$